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## ABJM theory in Batalin–Vilkovisky formulation

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## ABSTRACT

We analyze the quantum ABJM theory on  $\mathcal{N} = 1$  superspace in different gauges. We study the Batalin–Vilkovisky (BV) formulation for this model. By developing field/antifield dependent BRST transformation we establish connection between the two different solutions of the quantum master equation within the BV formulation.

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## 1. Introduction

The Aharony–Bergman–Jafferis–Maldacena (ABJM) theory is a conformal field theory in three-dimensional spacetime. The ABJM theory with gauge group  $U(N) \times U(N)$  is represented by  $N$   $M2$ -branes and has been constructed recently [1,2]. More precisely, it is shown that  $\mathcal{N} = 6$  supersymmetric Chern–Simons quiver gauge theory with bifundamental matter enjoying  $SO(4)$  flavor symmetry is dual to  $M$ -theory compactified on  $AdS_4 \times S^7/Z_k$ , and describes the low energy dynamics of a stack of  $M2$ -branes probing an orbifold singularity. This theory only has  $\mathcal{N} = 6$  supersymmetry but it is expected to be enhanced to the full  $\mathcal{N} = 8$  supersymmetry [3]. The  $M2$ -branes ending on  $M9$ -branes and gravitational waves have also been studied [4].

It may be noted that as the ABJM theory has gauge symmetry, it cannot be quantized without getting rid of these unphysical degrees of freedom. This can be done by fixing a gauge. The gauge fixing condition can be incorporated at a quantum level by adding ghost and gauge fixing terms to the original classical Lagrangian. It is known that for a gauge theory the new effective Lagrangian constructed as the sum of the original classical Lagrangian with the gauge fixing and the ghost terms, is invariant under a new set of transformations called the BRST transformations [5,6]. BRST symmetry has also been studied in non-linear gauges [7,8].

On the other hand the field/antifield formulation also known as the Batalin–Vilkovisky (BV) formalism [9–13] is one of the most

powerful techniques to study gauge field theories. The generalization of BRST by making the infinitesimal BRST parameter finite and field-dependent, known as FFBRST formulation [14] has many applications in gauge field theories [14–25]. Recently, we generalize the BRST symmetry by making the parameter field/antifield dependent for super-Chern–Simons theory [26]. We generalize such formulation in the case of ABJM theory on  $\mathcal{N} = 1$  superspace in the BV formalism.

In this work we discuss the ABJM theory from the perspective of gauge theory by discussing different gauge conditions. We investigate the different effective actions corresponding to the different gauge choices. We establish the BRST symmetry for the theory using two Grassmann parameters. Furthermore, the general BV quantization of the model has been analyzed. We generalize the BRST symmetry of the model by making the parameters field/antifield dependent. We compute the resulting Jacobian coming from the functional measure of the general generating functional. We find that for a particular choice of field/antifield dependent parameters (Eqs. (36) and (37)), the different gauges of ABJM theory can be connected. This result will be helpful to interrelate computations of physical quantities of the ABJM theory in linear and non-linear gauges.

The paper is presented in the following way. In Section 2, we analyze the classical ABJM theory in  $\mathcal{N} = 1$  superspace from the gauge symmetric point of view. Section 3 is devoted to describe the quantum analysis by studying different gauge conditions. The BV formalism is developed for ABJM theory in Section 4, which widens the quantization scheme. In Section 5, we develop a mapping between different solutions of extended quantum equation using the techniques of field/antifield dependent BRST symmetry. The results are summarized in the last section.

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## 2. The ABJM theory in $\mathcal{N} = 1$ superspace

We start with the Chern–Simons Lagrangian densities  $\mathcal{L}_{CS}$ ,  $\tilde{\mathcal{L}}_{CS}$  with gauge group's  $U(N)_k$  and  $U(N)_{-k}$  on  $\mathcal{N} = 1$  superspace defined by

$$\begin{aligned}\mathcal{L}_{CS} &= \frac{k}{2\pi} \int d^2\theta \text{Tr} \left[ \Gamma^a \omega_a + \frac{i}{3} [\Gamma^a, \Gamma^b] D_b \Gamma_a \right. \\ &\quad \left. + \frac{1}{3} [\Gamma^a, \Gamma^b] [\Gamma_a, \Gamma_b] \right]_|, \\ \tilde{\mathcal{L}}_{CS} &= \frac{k}{2\pi} \int d^2\theta \text{Tr} \left[ \tilde{\Gamma}^a \tilde{\omega}_a + \frac{i}{3} [\tilde{\Gamma}^a, \tilde{\Gamma}^b] D_b \tilde{\Gamma}_a \right. \\ &\quad \left. + \frac{1}{3} [\tilde{\Gamma}^a, \tilde{\Gamma}^b] [\tilde{\Gamma}_a, \tilde{\Gamma}_b] \right]_|, \end{aligned} \quad (1)$$

where  $k$  is an integer playing the role of a coupling constant.  $\omega_a$  and  $\tilde{\omega}_a$  have the following expression:

$$\begin{aligned}\omega_a &= \frac{1}{2} D^b D_a \Gamma_b - i [\Gamma^b, D_b \Gamma_a] - \frac{2}{3} [\Gamma^b, [\Gamma_b, \Gamma_a]], \\ \tilde{\omega}_a &= \frac{1}{2} D^b D_a \tilde{\Gamma}_b - i [\tilde{\Gamma}^b, D_b \tilde{\Gamma}_a] - \frac{2}{3} [\tilde{\Gamma}^b, [\tilde{\Gamma}_b, \tilde{\Gamma}_a]]. \end{aligned} \quad (2)$$

The  $D_a$  represents the super-derivative defined as

$$D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b, \quad (3)$$

and  $|$  means that the quantity is evaluated at  $\theta_a = 0$ . In component form the gauge connections  $\Gamma_a$  and  $\tilde{\Gamma}_a$  are expressed as

$$\begin{aligned}\Gamma_a &= \chi_a + B\theta_a + \frac{1}{2} (\gamma^\mu)_a A_\mu + i\theta^2 \left[ \lambda_a - \frac{1}{2} (\gamma^\mu \partial_\mu \chi)_a \right], \\ \tilde{\Gamma}_a &= \tilde{\chi}_a + \tilde{B}\theta_a + \frac{1}{2} (\gamma^\mu)_a \tilde{A}_\mu + i\theta^2 \left[ \tilde{\lambda}_a - \frac{1}{2} (\gamma^\mu \partial_\mu \tilde{\chi})_a \right]. \end{aligned} \quad (4)$$

The explicit expression for the Lagrangian density of the matter fields is given by

$$\begin{aligned}\mathcal{L}_M &= \frac{1}{4} \int d^2\theta \text{Tr} \left[ [\nabla_{(X)}^a X^{I\dagger} \nabla_{a(X)} X_I] \right. \\ &\quad \left. + [\nabla_{(Y)}^a Y^{I\dagger} \nabla_{a(Y)} Y_I] + \frac{16\pi}{k} \mathcal{V} \right]_|, \end{aligned} \quad (5)$$

where

$$\begin{aligned}\nabla_{(X)a} X^I &= D_a X^I + i\Gamma_a X^I - iX^I \tilde{\Gamma}_a, \\ \nabla_{(X)a} X^{I\dagger} &= D_a X^{I\dagger} + i\tilde{\Gamma}_a X^{I\dagger} - iX^{I\dagger} \Gamma_a, \\ \nabla_{(Y)a} Y^I &= D_a Y^I + i\tilde{\Gamma}_a Y^I - iY^I \Gamma_a, \\ \nabla_{(Y)a} Y^{I\dagger} &= D_a Y^{I\dagger} + i\Gamma_a Y^{I\dagger} - iY^{I\dagger} \tilde{\Gamma}_a. \end{aligned} \quad (6)$$

Now, the classical Lagrangian density for ABJM theory with the gauge group  $U(N) \times U(N)$  on  $\mathcal{N} = 1$  superspace is given by

$$\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS} - \tilde{\mathcal{L}}_{CS}, \quad (7)$$

which remains covariant under the following gauge transformations:

$$\begin{aligned}\delta \Gamma_a &= \nabla_a \xi, & \delta \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{\xi}, \\ \delta X^I &= i\xi X^I - iX^I \tilde{\xi}, & \delta X^{I\dagger} &= i\tilde{\xi} X^{I\dagger} - iX^{I\dagger} \xi, \\ \delta Y^I &= i\tilde{\xi} Y^I - iY^I \xi, & \delta Y^{I\dagger} &= i\xi Y^{I\dagger} - iY^{I\dagger} \tilde{\xi}, \end{aligned} \quad (8)$$

with the local parameters  $\xi$  and  $\tilde{\xi}$ . The super-covariant derivatives  $\nabla_a$  and  $\tilde{\nabla}_a$  are defined by

$$\nabla_a = D_a - i\Gamma_a, \quad \tilde{\nabla}_a = D_a - i\tilde{\Gamma}_a. \quad (9)$$

## 3. Gauge conditions and BRST symmetry

In this section, we investigate the quantum action for ABJM theory in linear and non-linear gauges. The nilpotency of BRST symmetry is also demonstrated for this theory.

### 3.1. Linear gauge

Being gauge invariant, the non-Abelian Chern–Simons theory on  $\mathcal{N} = 1$  superspace contains some redundant degrees of freedom. To quantize the theory correctly we need to choose a gauge. The covariant (Lorentz-type) gauge fixing conditions for ABJM theory are

$$G_1 \equiv D^a \Gamma_a = 0, \quad \tilde{G}_1 \equiv D^a \tilde{\Gamma}_a = 0. \quad (10)$$

These gauge fixing conditions can be incorporated in the theory at the quantum level by adding the following gauge fixing term to the original Lagrangian density,

$$\begin{aligned}\mathcal{L}_{gf} &= \int d^2\theta \text{Tr} \left[ ib_1 (D^a \Gamma_a) + \frac{\alpha}{2} b_1 b_1 \right. \\ &\quad \left. - i\tilde{b}_1 (D^a \tilde{\Gamma}_a) - \frac{\alpha}{2} \tilde{b}_1 \tilde{b}_1 \right]_|, \end{aligned} \quad (11)$$

where  $b_1$  and  $\tilde{b}_1$  are the Nakanishi–Lautrup auxiliary fields. The Faddeev–Popov ghost term corresponding to the above gauge fixing term is constructed as

$$\mathcal{L}_{gh} = \int d^2\theta \text{Tr} [i\tilde{c}_1 D^a \nabla_a c_1 - i\tilde{c}_1 D^a \tilde{\nabla}_a \tilde{c}_1]_|. \quad (12)$$

Now, we define the full quantum action for ABJM theory in Lorentz-type gauge by writing the gauge-fixing and the ghost terms collectively with classical action

$$\mathcal{L}_L = \mathcal{L}_c + \mathcal{L}_{gf} + \mathcal{L}_{gh}. \quad (13)$$

The BRST transformations, which leave the above effective action invariant, are written by

$$\begin{aligned}\delta_b \Gamma_a &= \nabla_a c_1 \Lambda, & \delta_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c}_1 \tilde{\Lambda}, \\ \delta_b c_1 &= -[c_1, c_1] \Lambda, & \delta_b \tilde{c}_1 &= -[\tilde{c}_1, \tilde{c}_1] \tilde{\Lambda}, \\ \delta_b \tilde{c}_1 &= b_1 \Lambda, & \delta_b \tilde{c}_1 &= \tilde{b}_1 \tilde{\Lambda}, \\ \delta_b b_1 &= 0, & \delta_b \tilde{b}_1 &= 0, \\ \delta_b X^I &= ic_1 X^I \Lambda - iX^I \tilde{c}_1 \tilde{\Lambda}, & \delta_b X^{I\dagger} &= i\tilde{c}_1 X^{I\dagger} \tilde{\Lambda} - iX^{I\dagger} c_1 \Lambda, \\ \delta_b Y^I &= i\tilde{c}_1 Y^I \tilde{\Lambda} - iY^I c_1 \Lambda, & \delta_b Y^{I\dagger} &= ic_1 Y^{I\dagger} \Lambda - iY^{I\dagger} \tilde{c}_1 \tilde{\Lambda}, \end{aligned} \quad (14)$$

where  $\Lambda$  and  $\tilde{\Lambda}$  are the infinitesimal anticommuting parameters of transformation.

### 3.2. Non-linear gauge

We start this subsection by demonstrating the ABJM theory in non-linear gauge as follows

$$\begin{aligned}\mathcal{L}_{NL} &= \mathcal{L}_c + \int d^2\theta \text{Tr} \left[ \frac{\alpha}{2} b_2^2 + ib_2 D^a \Gamma_a - iD^a \tilde{c}_2 \nabla_a c_2 \right. \\ &\quad - \frac{i}{2} D^a \Gamma_a [\tilde{c}_2, c_2] + \frac{\alpha}{8} [\tilde{c}_2, c_2]^2 - \frac{\alpha}{2} b_2 [\tilde{c}_2, c_2] + iD^a \tilde{c}_2 \nabla_a \tilde{c}_2 \\ &\quad - \frac{\alpha}{2} \tilde{b}_2^2 - i\tilde{b}_2 D^a \tilde{\Gamma}_a + \frac{i}{2} D^a \tilde{\Gamma}_a [\tilde{c}_2, \tilde{c}_2] \\ &\quad \left. - \frac{\alpha}{8} [\tilde{c}_2, \tilde{c}_2]^2 + \frac{\alpha}{2} \tilde{b}_2 [\tilde{c}_2, \tilde{c}_2] \right]_|. \end{aligned} \quad (15)$$

The above Lagrangian density can be obtained by performing the following shift in the Nakanishi–Lautrup auxiliary fields

$$b_1 \rightarrow b_2 - \frac{1}{2}[\bar{c}_2, c_2], \quad \tilde{b}_1 \rightarrow \tilde{b}_2 - \frac{1}{2}[\bar{\tilde{c}}_2, \tilde{c}_2]. \quad (16)$$

The BRST transformation under which the effective action in non-linear gauge remains invariant is given by

$$\begin{aligned} \delta_b \Gamma_a &= \nabla_a c_2 \Lambda, & \delta_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c}_2 \tilde{\Lambda}, \\ \delta_b c_2 &= -\frac{1}{2}[c_2, c_2] \Lambda, & \delta_b \tilde{c}_2 &= -\frac{1}{2}[\tilde{c}_2, \tilde{c}_2] \tilde{\Lambda}, \\ \delta_b \bar{c}_2 &= b_2 \Lambda - \frac{1}{2}[\bar{c}_2, c_2] \Lambda, & \delta_b \tilde{\bar{c}}_2 &= \tilde{b}_2 \tilde{\Lambda} - \frac{1}{2}[\tilde{\bar{c}}_2, \tilde{c}_2] \tilde{\Lambda}, \\ \delta_b b_2 &= -\frac{1}{2}[c_2, b_2] \Lambda - \frac{1}{8}[c_2, c_2, \bar{c}_2] \Lambda, \\ \delta_b \tilde{b}_2 &= -\frac{1}{2}[\tilde{c}_2, \tilde{b}_2] \tilde{\Lambda} - \frac{1}{8}[\tilde{c}_2, \tilde{c}_2, \tilde{\bar{c}}_2] \tilde{\Lambda}, \\ \delta_b X^I &= i c_2 X^I \Lambda - i X^I \bar{c}_2 \tilde{\Lambda}, & \delta_b X^{I\dagger} &= i \tilde{c}_2 X^{I\dagger} \tilde{\Lambda} - i X^{I\dagger} c_2 \Lambda, \\ \delta_b Y^I &= i \tilde{c}_2 Y^I \tilde{\Lambda} - i Y^I c_2 \Lambda, & \delta_b Y^{I\dagger} &= i c_2 Y^{I\dagger} \Lambda - i Y^{I\dagger} \tilde{c}_2 \tilde{\Lambda}. \end{aligned} \quad (17)$$

The effective action is also found invariant under the another set of BRST symmetry where roles of ghost and anti-ghost fields are interchanged, called as anti-BRST transformation and given by

$$\begin{aligned} \delta_{ab} \Gamma_a &= \nabla_a \bar{c}_2 \tilde{\Lambda}, & \delta_{ab} \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{\bar{c}}_2 \tilde{\tilde{\Lambda}}, \\ \delta_{ab} \bar{c}_2 &= -\frac{1}{2}[\bar{c}_2, \bar{c}_2] \tilde{\Lambda}, & \delta_{ab} \tilde{\bar{c}}_2 &= -\frac{1}{2}[\tilde{\bar{c}}_2, \tilde{\bar{c}}_2] \tilde{\tilde{\Lambda}}, \\ \delta_{ab} c_2 &= -b_2 \tilde{\Lambda} - \frac{1}{2}[\bar{c}_2, c_2] \tilde{\Lambda}, & \delta_{ab} \tilde{c}_2 &= -\tilde{b}_2 \tilde{\tilde{\Lambda}} - \frac{1}{2}[\tilde{\bar{c}}_2, \tilde{c}_2] \tilde{\tilde{\Lambda}}, \\ \delta_{ab} b_2 &= -\frac{1}{2}[\bar{c}_2, b_2] \tilde{\Lambda} + \frac{1}{8}[\bar{c}_2, \bar{c}_2, c_2] \tilde{\Lambda}, \\ \delta_{ab} \tilde{b}_2 &= -\frac{1}{2}[\tilde{\bar{c}}_2, \tilde{b}_2] \tilde{\tilde{\Lambda}} + \frac{1}{8}[\tilde{\bar{c}}_2, \tilde{\bar{c}}_2, \tilde{c}_2] \tilde{\tilde{\Lambda}}, \\ \delta_{ab} X^I &= i \tilde{c}_2 X^I \tilde{\Lambda} - i X^I \bar{c}_2 \tilde{\tilde{\Lambda}}, & \delta_{ab} X^{I\dagger} &= i \tilde{\bar{c}}_2 X^{I\dagger} \tilde{\tilde{\Lambda}} - i X^{I\dagger} \bar{c}_2 \tilde{\Lambda}, \\ \delta_{ab} Y^I &= i \tilde{\bar{c}}_2 Y^I \tilde{\tilde{\Lambda}} - i Y^I \bar{c}_2 \tilde{\Lambda}, & \delta_{ab} Y^{I\dagger} &= i \bar{c}_2 Y^{I\dagger} \tilde{\Lambda} - i Y^{I\dagger} \tilde{\bar{c}}_2 \tilde{\tilde{\Lambda}}. \end{aligned} \quad (18)$$

The above BRST and anti-BRST transformations are nilpotent as well as absolutely anticommuting, i.e.

$$\delta_b^2 = 0, \quad \delta_{ab}^2 = 0, \quad \delta_b \delta_{ab} + \delta_{ab} \delta_b = 0. \quad (19)$$

The gauge-fixing and ghost terms of the ABJM model in non-linear gauge can be expressed in terms of BRST and anti-BRST exact terms as follows

$$\begin{aligned} \mathcal{L}_{NL} &= \frac{i}{2} \delta_b \delta_{ab} \int d^2\theta \text{Tr}[\Gamma_a \Gamma^a - \tilde{\Gamma}_a \tilde{\Gamma}^a - i \alpha \bar{c}_2 c_2 + i \alpha \tilde{\bar{c}}_2 \tilde{c}_2] \\ &= -\frac{i}{2} \delta_{ab} \delta_b \int d^2\theta \text{Tr}[\Gamma_a \Gamma^a - \tilde{\Gamma}_a \tilde{\Gamma}^a - i \alpha \bar{c}_2 c_2 + i \alpha \tilde{\bar{c}}_2 \tilde{c}_2]. \end{aligned} \quad (20)$$

In the next section we analyze the theory in BV formulation.

#### 4. ABJM theory in BV formulation

To establish the theory in BV formulation we need to introduce antifields corresponding to fields with opposite statistics. In terms of fields/antifields, the generating functional for the ABJM theory in Lorentz-type gauge is

$$\begin{aligned} Z_L &= \int \mathcal{D}\Phi e^{iW_L[\Phi, \Phi^*, \bar{\Phi}, \bar{\Phi}^*]} \\ &= \int \mathcal{D}\Phi \exp \left[ i \int dv \left( \mathcal{L}_c + \int d^2\theta \text{Tr}[\Gamma^{a*} \nabla_a c_1 \right. \right. \\ &\quad \left. \left. + \tilde{\Gamma}^{a*} \tilde{\nabla}_a \tilde{c}_1 + \bar{c}_1^* b_1 + \tilde{\bar{c}}_1^* \tilde{b}_1 \right) \right], \end{aligned} \quad (21)$$

where  $W_L$  is the extended quantum action and integration  $\int dv$  refers to  $\int d^3x$ . The gauge-fixed fermion for ABJM theory in Lorentz gauge is defined by

$$\Psi_L = \bar{c}_1 \left( i D^a \Gamma_a + \frac{\alpha}{2} b_1 \right) - \tilde{\bar{c}}_1 \left( i D^a \tilde{\Gamma}_a + \frac{\alpha}{2} \tilde{b}_1 \right). \quad (22)$$

With the help of this gauge-fixed fermion we compute the anti-fields for the Lorentz gauge as following:

$$\begin{aligned} X^{I*} &= \frac{\delta \Psi_L}{\delta X^I} = 0, & X^{I\dagger*} &= \frac{\delta \Psi_L}{\delta X^{I\dagger}} = 0, & Y^{I*} &= \frac{\delta \Psi_L}{\delta Y^I} = 0, \\ Y^{I\dagger*} &= \frac{\delta \Psi_L}{\delta Y^{I\dagger}} = 0, & c_1^* &= \frac{\delta \Psi_L}{\delta c_1} = 0, & \tilde{c}_1^* &= \frac{\delta \Psi_L}{\delta \tilde{c}_1} = 0, \\ \Gamma^{a*} &= \frac{\delta \Psi_L}{\delta \Gamma_a} = -i D^a \bar{c}_1, & \tilde{\Gamma}^{a*} &= \frac{\delta \Psi_L}{\delta \tilde{\Gamma}_a} = i D^a \tilde{\bar{c}}_1, \\ \bar{c}_1^* &= \frac{\delta \Psi_L}{\delta \bar{c}_1} = i D^a \Gamma_a + \frac{\alpha}{2} b_1, & \tilde{\bar{c}}_1^* &= \frac{\delta \Psi_L}{\delta \tilde{\bar{c}}_1} = -i D^a \tilde{\Gamma}_a - \frac{\alpha}{2} \tilde{b}_1. \end{aligned} \quad (23)$$

However, the generating functional for ABJM in the non-linear gauge in terms of fields/antifields is given by

$$\begin{aligned} Z_{NL} &= \int \mathcal{D}\Phi e^{iW_{NL}[\Phi, \Phi^*, \bar{\Phi}, \bar{\Phi}^*]} \\ &= \int \mathcal{D}\Phi \exp \left[ i \int dv \left( \mathcal{L}_c + \int d^2\theta \text{Tr} \left[ \Gamma^{a*} \nabla_a c_2 + \tilde{\Gamma}^{a*} \tilde{\nabla}_a \tilde{c}_2 \right. \right. \right. \\ &\quad \left. \left. + \bar{c}_2^* \left( b_2 - \frac{1}{2}[\bar{c}_2, c_2] \right) + \tilde{\bar{c}}_2^* \left( \tilde{b}_2 - \frac{1}{2}[\tilde{\bar{c}}_2, \tilde{c}_2] \right) \right] \right) \right]. \end{aligned} \quad (24)$$

We evaluate the expression for the gauge-fixing fermion for the non-linear gauge as following:

$$\begin{aligned} \Psi_{NL} &= \bar{c}_2 \left( i D^a \Gamma_a + \frac{\alpha}{2} b_2 - \frac{\alpha}{4}[\bar{c}_2, c_2] \right) \\ &\quad - \tilde{\bar{c}}_2 \left( i D^a \tilde{\Gamma}_a + \frac{\alpha}{2} \tilde{b}_2 - \frac{\alpha}{4}[\tilde{\bar{c}}_2, \tilde{c}_2] \right). \end{aligned} \quad (25)$$

The antifields in this case are identified as

$$\begin{aligned} X^{I*} &= \frac{\delta \Psi_{NL}}{\delta X^I} = 0, & X^{I\dagger*} &= \frac{\delta \Psi_{NL}}{\delta X^{I\dagger}} = 0, & Y^{I*} &= \frac{\delta \Psi_{NL}}{\delta Y^I} = 0, \\ Y^{I\dagger*} &= \frac{\delta \Psi_{NL}}{\delta Y^{I\dagger}} = 0, & c_2^* &= \frac{\delta \Psi_{NL}}{\delta c_2} = 0, & \tilde{c}_2^* &= \frac{\delta \Psi_{NL}}{\delta \tilde{c}_2} = 0, \\ \Gamma^{a*} &= \frac{\delta \Psi_{NL}}{\delta \Gamma_a} = -i D^a \bar{c}_2, & \tilde{\Gamma}^{a*} &= \frac{\delta \Psi_{NL}}{\delta \tilde{\Gamma}_a} = i D^a \tilde{\bar{c}}_2, \\ \bar{c}_2^* &= \frac{\delta \Psi_{NL}}{\delta \bar{c}_2} = i D^a \Gamma_a + \frac{\alpha}{2} b_2 - \frac{\alpha}{4}[\bar{c}_2, c_2], \\ \tilde{\bar{c}}_2^* &= \frac{\delta \Psi_{NL}}{\delta \tilde{\bar{c}}_2} = -i D^a \tilde{\Gamma}_a - \frac{\alpha}{2} \tilde{b}_2 + \frac{\alpha}{4}[\tilde{\bar{c}}_2, \tilde{c}_2]. \end{aligned} \quad (26)$$

We note the difference between the two extended quantum actions as follows,

$$\begin{aligned}
W_{NL} - W_L = \int d\nu \int d^2\theta \text{Tr} \Big[ & \left( -iD^a \bar{c}_2 \nabla_a c_2 \right. \\
& + iD^a \bar{c}_1 \nabla_a c_1 + iD^a \Gamma_a (b_2 - b_1) - \frac{i}{2} D^a \Gamma_a [\bar{c}_2, c_2] \\
& + \frac{\alpha}{2} (b_2^2 - b_1^2) + \frac{\alpha}{8} [\bar{c}_2, c_2]^2 - \frac{\alpha}{2} b_2 [\bar{c}_2, c_2] \Big) \\
& + \left( iD^a \bar{c}_2 \nabla_a \tilde{c}_2 - iD^a \tilde{c}_1 \nabla_a \bar{c}_1 \right. \\
& - iD^a \tilde{\Gamma}_a (\tilde{b}_2 - \tilde{b}_1) + \frac{i}{2} D^a \tilde{\Gamma}_a [\tilde{c}_2, \tilde{c}_2] - \frac{\alpha}{2} (\tilde{b}_2^2 - \tilde{b}_1^2) \\
& \left. \left. - \frac{\alpha}{8} [\tilde{c}_2, \tilde{c}_2]^2 + \frac{\alpha}{2} \tilde{b}_2 [\tilde{c}_2, \tilde{c}_2] \right) \right]. \quad (27)
\end{aligned}$$

The extended quantum actions,  $W_\psi[\Phi, \Phi^*] \equiv (W_{NL}, W_L)$ , satisfy certain rich mathematical relation so-called quantum master equation, which is given by

$$\Delta e^{iW_\psi[\Phi, \Phi^*]} = 0, \quad \Delta \equiv \frac{\partial_r}{\partial \Phi^*} \frac{\partial_l}{\partial \Phi} (-1)^{\epsilon+1}. \quad (28)$$

Here we note that the extended quantum actions  $W_{NL}$  and  $W_L$  are two different possible solutions of the quantum master equation.

In the next section, our goal would be to establish a map between the two generating functionals corresponding to the above extended actions using the technique of field/antifield dependent BRST transformations.

## 5. A mapping between solutions of quantum master equation

We first analyze the field/antifield dependent BRST transformation which is characterized by the field/antifield dependent BRST parameter. To achieve the goal, we define the usual BRST transformation for the generic fields  $\Phi_\alpha(x)$  and  $\tilde{\Phi}_\alpha(x)$  written compactly as

$$\begin{aligned}
\Phi'_\alpha(x) - \Phi_\alpha(x) &= \delta_b \Phi_\alpha(x) = s_b \Phi_\alpha(x) \Lambda = \mathcal{R}_\alpha(x) \Lambda, \\
\tilde{\Phi}'_\alpha(x) - \tilde{\Phi}_\alpha(x) &= \delta_b \tilde{\Phi}_\alpha(x) = s_b \tilde{\Phi}_\alpha(x) \tilde{\Lambda} = \tilde{\mathcal{R}}_\alpha(x) \tilde{\Lambda}, \quad (29)
\end{aligned}$$

where  $\mathcal{R}_\alpha(x)(s_b \Phi_\alpha(x))$  and  $\tilde{\mathcal{R}}_\alpha(x)(s_b \tilde{\Phi}_\alpha(x))$  are the Slavnov variations of the field  $\Phi_\alpha(x)$  and  $\tilde{\Phi}_\alpha(x)$  satisfying  $\delta_b \mathcal{R}_\alpha(x) = \delta_b \tilde{\mathcal{R}}_\alpha(x) = 0$ . Here the infinitesimal transformation parameters  $\Lambda$  and  $\tilde{\Lambda}$  are the Grassmann parameters and don't depend on any field/antifield.

Now, we present the field/antifield dependent BRST transformation as follows

$$\begin{aligned}
\delta_b \Phi_\alpha(x) &= \Phi'_\alpha(x) - \Phi_\alpha(x) = \mathcal{R}_\alpha(x) \Lambda[\Phi, \Phi^*], \\
\delta_b \tilde{\Phi}_\alpha(x) &= \tilde{\Phi}'_\alpha(x) - \tilde{\Phi}_\alpha(x) = \tilde{\mathcal{R}}_\alpha(x) \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*], \quad (30)
\end{aligned}$$

where the Grassmann parameters  $\Lambda[\Phi, \Phi^*]$  and  $\tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*]$  depend on the field/antifield explicitly. The field/antifield dependent BRST transformation for the ABJM theory is constructed by making the transformation parameter of (14) and (17) field/antifield dependent. Though being symmetry of the extended action such field/antifield dependent transformation is not nilpotent any more. We notice that under such transformation the path integral measure of generating functional changes non-trivially. We compute the change in the generating functional as follows,

$$\begin{aligned}
\delta_b Z_L &= \int \mathcal{D}\Phi (s\text{Det } J[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*]) e^{iW_L[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*]} \\
&= \int \mathcal{D}\Phi e^{i(W_L[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*] - i s\text{Tr} \ln J[\Phi, \Phi^*])}. \quad (31)
\end{aligned}$$

Furthermore, the Jacobian matrix appearing above for the field/antifield dependent BRST transformation is given by

$$\begin{aligned}
J_{\alpha\beta} &= [\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*] \\
&= \frac{(\delta \Phi'_\alpha, \delta \tilde{\Phi}'_\alpha)}{(\delta \Phi_\beta, \delta \tilde{\Phi}_\beta)} \\
&= \delta_{\alpha\beta} + \frac{\delta \mathcal{R}_\alpha(x)}{\delta \Phi_\beta} \Lambda[\Phi, \Phi^*] + \mathcal{R}_\alpha(x) \frac{\delta \Lambda[\Phi, \Phi^*]}{\delta \Phi_\beta} \\
&\quad + \frac{\delta \tilde{\mathcal{R}}_\alpha(x)}{\delta \tilde{\Phi}_\beta} \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*] + \tilde{\mathcal{R}}_\alpha(x) \frac{\delta \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*]}{\delta \tilde{\Phi}_\beta}. \quad (32)
\end{aligned}$$

Utilizing (32) and the nilpotency of the BRST transformation (i.e.  $s_b^2 = 0$ ) we obtain the following relation [27,28]

$$\begin{aligned}
s\text{Tr} \ln J[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*] \\
= -\ln(1 + s_b \Lambda[\Phi, \Phi^*] + s_b \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*]). \quad (33)
\end{aligned}$$

Because of the anticommuting nature of  $\Lambda[\Phi, \Phi^*]$  the determinant simplifies to

$$s\text{Det } J[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*] = \frac{1}{1 + s_b \Lambda[\Phi, \Phi^*] + s_b \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*]}. \quad (34)$$

Plugging this value of determinant in the relation (31) we get

$$\begin{aligned}
s_b Z_L &= \int \mathcal{D}\Phi \exp(iW_L[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*] - \ln(1 + s_b \Lambda[\Phi, \Phi^*] \\
&\quad + s_b \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*])). \quad (35)
\end{aligned}$$

This is a very general expression for the change in the generating functional of the ABJM theory under field/antifield dependent BRST transformation because it involves an arbitrary  $\Lambda[\Phi, \Phi^*]$ . Now we evaluate such variation under a specific choice of the field/antifield dependent transformation parameters chosen as follows

$$\begin{aligned}
\Lambda[\Phi, \Phi^*] &= \int d\nu \int d^2\theta \psi (s_b \psi)^{-1} (\exp\{-is_b \psi\} - 1)_|, \\
\tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*] &= \int d\nu \int d^2\theta \tilde{\psi} (s_b \tilde{\psi})^{-1} (\exp\{-is_b \tilde{\psi}\} - 1)_|, \quad (36)
\end{aligned}$$

where  $\psi$  and  $\tilde{\psi}$  are defined by

$$\begin{aligned}
\psi &= (\bar{c}_2 \bar{c}_2^* - \bar{c}_1 \bar{c}_1^*), \\
\tilde{\psi} &= (\bar{\tilde{c}}_2 \bar{\tilde{c}}_2^* - \bar{\tilde{c}}_1 \bar{\tilde{c}}_1^*). \quad (37)
\end{aligned}$$

We now demonstrate that the above choices of  $\Lambda$  and  $\tilde{\Lambda}$  relate the two generating functionals (21) and (24). This is one of the main results of this paper.

The Jacobian expression (33) for the above choice of parameter yields

$$\begin{aligned}
i \ln(1 + s_b \Lambda[\Phi, \Phi^*] + s_b \tilde{\Lambda}[\tilde{\Phi}, \tilde{\Phi}^*]) \\
= \int d\nu \int d^2\theta (s_b \psi + s_b \tilde{\psi}) \\
= \int d\nu \int d^2\theta [(s_b \bar{c}_2) \bar{c}_2^* - (s_b \bar{c}_1) \bar{c}_1^* + \bar{c}_2 (s_b \bar{\tilde{c}}_2^*) - \bar{c}_1 (s_b \bar{\tilde{c}}_1^*) \\
+ (s_b \bar{\tilde{c}}_2) \bar{\tilde{c}}_2^* - (s_b \bar{\tilde{c}}_1) \bar{\tilde{c}}_1^* + \bar{\tilde{c}}_2 (s_b \bar{\tilde{c}}_2^*) - \bar{\tilde{c}}_1 (s_b \bar{\tilde{c}}_1^*)]. \quad (38)
\end{aligned}$$

Now we can use the antifield expressions (23), (26) and the linear and non-linear BRST transformations (14), (17) to complete the computation. There are eight terms in the parentheses, let us calculate some of them. Firstly, we calculate

$$\begin{aligned}
(s_b \bar{c}_2) \bar{c}_2^* &= \left( b_2 - \frac{1}{2} [\bar{c}_2, c_2] \right) \left( iD^a \Gamma_a + \frac{\alpha}{2} b_2 - \frac{\alpha}{4} [\bar{c}_2, c_2] \right) \\
&= b_2 \left( iD^a \Gamma_a + \frac{\alpha}{2} b_2 \right) - \frac{i}{2} D^a \Gamma_a [\bar{c}_2, c_2] + \frac{\alpha}{8} [\bar{c}_2, c_2]^2 \\
&\quad - \frac{\alpha}{2} b_2 [\bar{c}_2, c_2].
\end{aligned} \quad (39)$$

The second term leads to

$$(s_b \bar{c}_1) \bar{c}_1^* = b_1 \left( iD^a \Gamma_a + \frac{\alpha}{2} b_1 \right). \quad (40)$$

However, the third term is computed as

$$\bar{c}_2 (s_b \bar{c}_2^*) = \bar{c}_2 \left( iD^a s_b \Gamma_a + \frac{\alpha}{2} s_b b_2 + \frac{\alpha}{4} s_b [\bar{c}_2, c_2] \right). \quad (41)$$

Now, utilizing the Slavnov variation of (17) we have

$$\begin{aligned}
s_b \bar{c}_2 &= b_2 - \frac{1}{2} [\bar{c}_2, c_2], \\
\text{or, } s_b^2 \bar{c}_2 &= 0 = s_b b_2 - \frac{1}{2} s_b [\bar{c}_2, c_2], \\
\text{or, } s_b [\bar{c}_2, c_2] &= 2s_b b_2.
\end{aligned} \quad (42)$$

Putting the values of (42) back in (41) gives

$$\begin{aligned}
\bar{c}_2 (s_b \bar{c}_2^*) &= \bar{c}_2 \left( iD^a s_b \Gamma_a + \frac{\alpha}{2} s_b b_2 - \frac{\alpha}{4} s_b [\bar{c}_2, c_2] \right), \\
&= \bar{c}_2 \left( iD^a s_b \Gamma_a + \frac{\alpha}{2} s_b b_2 - \frac{\alpha}{4} (2s_b b_2) \right), \\
&= \bar{c}_2 iD^a \nabla_a c_2 = -iD^a \bar{c}_2 \nabla_a c_2.
\end{aligned} \quad (43)$$

The fourth term is calculated by

$$\bar{c}_1 s_b \bar{c}_1^* = \bar{c}_1 iD^a \nabla_a c_1 = -iD^a \bar{c}_1 \nabla_a c_1. \quad (44)$$

Putting together (39), (40), (43) and (44) we obtain the following expression

$$\begin{aligned}
(s_b \bar{c}_2) \bar{c}_2^* - (s_b \bar{c}_1) \bar{c}_1^* + \bar{c}_2 (s_b \bar{c}_2^*) - \bar{c}_1 (s_b \bar{c}_1^*) \\
&= -iD^a \bar{c}_2 \nabla_a c_2 + iD^a \bar{c}_1 \nabla_a c_1 + iD^a \Gamma_a (b_2 - b_1) \\
&\quad - \frac{i}{2} D^a \Gamma_a [\bar{c}_2, c_2] + \frac{\alpha}{2} (b_2^2 - b_1^2) + \frac{\alpha}{8} [\bar{c}_2, c_2]^2 \\
&\quad - \frac{\alpha}{2} b_2 [\bar{c}_2, c_2].
\end{aligned} \quad (45)$$

Following a similar computation we have for

$$\begin{aligned}
(s_b \bar{c}_2) \bar{c}_2^* - (s_b \bar{c}_1) \bar{c}_1^* + \bar{c}_2 (s_b \bar{c}_2^*) - \bar{c}_1 (s_b \bar{c}_1^*) \\
&= iD^a \bar{c}_2 \nabla_a \bar{c}_2 - iD^a \bar{c}_1 \nabla_a \bar{c}_1 - iD^a \tilde{\Gamma}_a (\tilde{b}_2 - \tilde{b}_1) \\
&\quad + \frac{i}{2} D^a \tilde{\Gamma}_a [\tilde{c}_2, \tilde{c}_2] - \frac{\alpha}{2} (\tilde{b}_2^2 - \tilde{b}_1^2) - \frac{\alpha}{8} [\tilde{c}_2, \tilde{c}_2]^2 \\
&\quad + \frac{\alpha}{2} \tilde{b}_2 [\tilde{c}_2, \tilde{c}_2].
\end{aligned} \quad (46)$$

Therefore, it is easy to see from Eqs. (27), (35), (38), (45) and (46) that

$$\delta_b Z_L = Z_{NL}. \quad (47)$$

Hence we have shown that under field/antifield dependent BRST transformation with the appropriate choice of parameters (36) and (37), the different solutions of the quantum master equation can be related.

## 6. Conclusion

In this paper we have established the ABJM theory at quantum level by investigating it in the BV formulation on  $\mathcal{N} = 1$  superspace. For this purpose, we have extended the configuration space by introducing the antifields corresponding to the fields of ABJM model. Further, we have calculated the exact values of antifields by choosing the suitable gauge-fixing fermion. We have mainly discussed the Lorentz-type and Curci–Ferrari type gauges from the BRST quantization perspectives. The quantum master equation for the ABJM theory, having different possible solutions, is also established. Furthermore, we have generalized the BRST symmetry of the theory by developing the field/antifield dependent parameters. Here we need two parameters of transformation rather than one. We have also successfully demonstrated how a particular choice of the transformation parameters can relate two different generating functionals in the Lorentz-type and the Curci–Ferrari type gauges.

Our analysis on BV formulation of ABJM theory will provide a convenient way to study the possible violations of the symmetries of the action by quantum effects. Such analysis may also be useful in calculating the S-matrix of the theory because we have already computed the definite values of antifields. The master equation discussed above is more fundamental than the Zinn–Justin equation which guarantees the renormalizability of the ABJM theory, since the master equation relies on the fundamental action rather than the quantum effective action. The present investigation is a step towards the study of the deformations of the action and anomalies.

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